PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Multipole Mixtures in the Mössbauer Effect*

J. T. Dehn, J. G. Marzolf, and J. F. Salmon
RINS, Woodstock College, Woodstock, Maryland
(Received 15 May 1964)

A theory of multipole mixtures applicable to resonance absorption is developed. The method used is an extension of Malus' law to include elliptically polarized multipole mixtures. The case of dipole-quadrupole mixtures is treated in detail as a means of measuring $E2/M1$ mixing ratios and checking time-reversal invariance in certain nuclei.

INTRODUCTION

RECENTLY, Frauenfelder[1] and his co-workers have developed and applied the theory of elliptical polarization in the Mössbauer effect. In their paper (referred to hereafter as F) a method of complex vector parameterization was used to derive an expression for the transmission pattern,

$$I\Sigma = II' \cos^2\Theta\,, \tag{1}$$

such that the factor I, which is proportional to the intensity of the emitted radiation, can be factored out on the right side of Eq. (1). Then Σ, which is proportional to the absorption cross section, may be found explicitly in terms of the Euler angles α, β for the oriented emitter and α', β' for the oriented absorber. The angular factor, $\cos^2\Theta$, is given in Table V of F for pure dipole and quadrupole radiation and for various values of M and M', the changes in magnetic quantum number for emitter and absorber, respectively. However, this method proved too complicated for a convenient treatment of multipole mixtures. It is the purpose of the present paper to develop the theory of multipole mixtures by a more direct method.

THEORY

Let us begin with a modified form of Eq. (38) in F for the electric field vector,

$$\mathcal{E}_{21}(M) = a\mathbf{E}_2(M) + be^{i\varphi}\mathbf{E}_1(M)\,, \tag{2}$$

where a and $be^{i\varphi}$ are products of the appropriate Wigner $3j$ symbols and reduced matrix elements χ, as explained by F. We can choose a, b, and the relative phase of the nuclear matrix elements φ to be real numbers. The angular dependence is expressed by the vector

$$\mathbf{E}_L(M) = e^{iM\gamma}\left(\hat{\eta}_1 e^{i\alpha} d_{1M}^{(L)}(\beta) \pm \hat{\eta}_{-1} e^{-i\alpha} d_{-1M}^{(L)}(\beta)\right)\,, \tag{3}$$

where the upper signs refer to electric radiation and the lower signs to magnetic radiation. We shall be concerned with the case of most physical interest, namely, a mixture of magnetic dipole ($L=1$) and electric quadrupole ($L=2$) radiation. In the last equation, γ is the Euler angle measured about the axis of nuclear orientation, β is the angle between this axis and the axis of observation, while α is the azimuthal angle measured about the axis of observation. The complex unit vectors are

$$\hat{\eta}_{\pm 1} = \mp(\hat{\imath} \pm i\hat{\jmath})/\sqrt{2} \tag{4}$$

so that $\hat{\eta}_\mu^* \cdot \hat{\eta}_\nu = \delta_{\mu\nu}$, while the rotation matrix elements $d_{\mu M}^{(L)}$ are given in Table II of F, reproduced here as Table I.

The quantity we are interested in is

$$\begin{aligned} I\Sigma &= |\mathcal{E}_{21}^*(M)\cdot\mathcal{E}_{21}'(M')|^2 \\ &= |aa'\mathbf{E}_2^*(M)\cdot\mathbf{E}_2'(M') + ab'e^{i\varphi'}\mathbf{E}_2^*(M)\cdot\mathbf{E}_1'(M') \\ &\quad + a'be^{-i\varphi}\mathbf{E}_1^*(M)\cdot\mathbf{E}_2'(M') \\ &\quad + bb'e^{-i(\varphi-\varphi')}\mathbf{E}_1^*(M)\cdot\mathbf{E}_1'(M')|^2. \end{aligned} \tag{5}$$

As a first step it is convenient to compute the complex numbers listed in Table II by using Eq. (3). From Table II and Eq. (5) with a or b set equal to zero, we can derive the formulas given in F. For example, with $b=b'=0$, we may compute $I\Sigma = |\mathcal{E}_2^*(2)\cdot\mathcal{E}_2'(\pm 2)|^2$ which appears as the last entry in Table III. If in addition we let $a=a'$, $\alpha=\alpha'$, $\beta=\beta'$, and use the upper signs in this formula, we find the intensity squared and finally the intensity

$$I = \tfrac{1}{8}a^2(4\sin^2\beta + \sin^2 2\beta) = \tfrac{1}{2}a^2\sin^2\beta(1+\cos^2\beta)\,. \tag{6}$$

We may divide by this quantity to find

$$\Sigma = (I)^{-1}|\mathcal{E}^*\cdot\mathcal{E}'|^2 = I'\cos^2\Theta \tag{7}$$

and divide again by I' to find

$$\cos^2\Theta = (II')^{-1}|\mathcal{E}^*\cdot\mathcal{E}'|^2\,, \tag{8}$$

which appears as the first (or fourth) entry in Table V of F. We have omitted subscripts and arguments in Eqs. (7) and (8) to indicate their general applicability even to multipole mixtures.

Table I. Reduced rotation matrix elements, $d_{\mu M}^{(L)}$, for dipole and quadrupole cases.

μ	M	$d_{\mu M}^{(1)}$	$d_{\mu M}^{(2)}$
± 1	± 2	$\cdots$	$\mp(2\sin\beta+\sin 2\beta)/4$
± 1	± 1	$\cos^2\beta/2$	$(\cos\beta+\cos 2\beta)/2$
± 1	0	$\pm(1/\sqrt{2})\sin\beta$	$\pm(3/8)^{1/2}\sin 2\beta$
± 1	∓ 1	$\sin^2\beta/2$	$(\cos\beta-\cos 2\beta)/2$
± 1	∓ 2	$\cdots$	$\pm(2\sin\beta-\sin 2\beta)/4$

* This work was supported in part by the National Aeronautics and Space Administration under Research Grant NsG 670.

[1] H. Frauenfelder, D. E. Nagle, R. D. Taylor, D. R. F. Cochran, and W. M. Visscher, Phys. Rev. **126**, 1065 (1962).

TABLE II. Complex numbers $\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(M')$.

M	M'	$\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(M')$
		(a) Dipole radiation ($s=t=1$)
1	±1	$\frac{1}{2}e^{-i(\gamma\mp\gamma')}[(1\pm\cos\beta\cos\beta')\cos(\alpha-\alpha')-i(\cos\beta\pm\cos\beta')\sin(\alpha-\alpha')]$
±1	0	$(1/\sqrt{2})e^{\mp i\gamma}\sin\beta'[\pm\cos\beta\cos(\alpha-\alpha')-i\sin(\alpha-\alpha')]$
0	0	$\sin\beta\sin\beta'\cos(\alpha-\alpha')$
		(b) Quadrupole radiation ($s=t=2$)
2	±2	$\pm\frac{1}{2}e^{-i2(\gamma\mp\gamma')}\sin\beta\sin\beta'[(1\pm\cos\beta\cos\beta')\cos(\alpha-\alpha')-i(\cos\beta\pm\cos\beta')\sin(\alpha-\alpha')]$
2	±1	$-\frac{1}{2}e^{-i(2\gamma\mp\gamma')}\sin\beta[(\cos\beta'\pm\cos\beta\cos2\beta')\cos(\alpha-\alpha')-i(\cos\beta\cos\beta'\pm\cos2\beta')\sin(\alpha-\alpha')]$
±2	0	$\mp(3/8)^{1/2}e^{\mp i2\gamma}\sin\beta\sin2\beta'[\pm\cos\beta\cos(\alpha-\alpha')-i\sin(\alpha-\alpha')]$
1	±1	$\frac{1}{2}e^{-i(\gamma\mp\gamma')}[(\cos\beta\cos\beta'\pm\cos2\beta\cos2\beta')\cos(\alpha-\alpha')-i(\cos\beta'\cos2\beta\pm\cos\beta\cos2\beta')\sin(\alpha-\alpha')]$
±1	0	$(3/8)^{1/2}e^{\mp i\gamma}\sin2\beta'[\pm\cos2\beta\cos(\alpha-\alpha')-i\cos\beta\sin(\alpha-\alpha')]$
0	0	$\frac{3}{4}\sin2\beta\sin2\beta'\cos(\alpha-\alpha')$
		(c) Cross terms ($s=1, t=2$)
1	±2	$\mp\frac{1}{2}e^{-i(\gamma\mp2\gamma')}\sin\beta'[(\cos\beta\pm\cos\beta')\cos(\alpha-\alpha')-i(1\pm\cos\beta\cos\beta')\sin(\alpha-\alpha')]$
1	±1	$\frac{1}{2}e^{-i(\gamma\mp\gamma')}[(\cos\beta\cos\beta'\pm\cos2\beta')\cos(\alpha-\alpha')-i(\cos\beta'\pm\cos\beta\cos2\beta')\sin(\alpha-\alpha')]$
±1	0	$(3/8)^{1/2}e^{\mp i\gamma}\sin2\beta'[\cos(\alpha-\alpha')\mp i\cos\beta\sin(\alpha-\alpha')]$
0	±2	$\mp(1/\sqrt{2})e^{\pm i2\gamma'}\sin\beta\sin\beta'[\cos(\alpha-\alpha')\mp i\cos\beta'\sin(\alpha-\alpha')]$
0	±1	$(1/\sqrt{2})e^{\pm i\gamma'}\sin\beta[\cos\beta'\cos(\alpha-\alpha')\mp i\cos2\beta'\sin(\alpha-\alpha')]$
0	0	$-i(\sqrt{3}/2)\sin\beta\sin2\beta'\sin(\alpha-\alpha')$

Before proceeding further, let us derive two auxiliary formulas which may be used to fill out Table II. The first is hardly more than a rearrangement of the quantities involved, as follows:

$$\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(M')=[\mathbf{E}_s(M)\cdot\mathbf{E}_t'^*(M')]^* =[\mathbf{E}_t^*(M')\cdot\mathbf{E}_s'(M)]_x^*, \quad (9)$$

where subscript x denotes the operation $\alpha\leftrightarrow\alpha'$, $\beta\leftrightarrow\beta'$, $\gamma\leftrightarrow\gamma'$. This formula is convenient for finding such quantities as $\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(M')$ from $\mathbf{E}_t^*(M')\cdot\mathbf{E}_s'(M)$. The second formula is

$$\mathbf{E}_s^*(-M)\cdot\mathbf{E}_t'(\mp M') =(-1)^n[\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(\pm M')]^*, \quad (10)$$

where $n=M+M'+(1-\delta_{st})$ and δ_{st} is the Kronecker delta. It is useful for obtaining $\mathbf{E}_s^*(-M)\cdot\mathbf{E}_t'(\mp M')$ from $\mathbf{E}_s^*(M)\cdot\mathbf{E}_t'(\pm M')$. We may prove it by using Eq. (3) in Eq. (10) and equating the coefficients of the exponentials in the resulting expression. Thus we must show that

$$d_{1-M}{}^{(s)}d_{1\mp M'}{}'^{(t)}=(-1)^{M+M'}d_{-1M}{}^{(s)}d_{-1\pm M'}{}'^{(t)} \quad (11)$$

and

$$d_{1M}{}^{(s)}d_{1\pm M'}{}'^{(t)}=(-1)^{M+M'}d_{-1-M}{}^{(s)}d_{-1\mp M'}{}'^{(t)}, \quad (12)$$

where we have ignored the symbol for the complex conjugate since the $d_{\mu M}{}^{(L)}$ are real. These relations follow from the equation

$$d_{1\pm M}{}^{(L)}=(-1)^{1+M}d_{-1\mp M}{}^{(L)}, \quad (13)$$

which can be obtained from an examination of Table I or, in the general case, from Eq. (4.19) of Rose.[2]

[2] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 54.

By using Eqs. (9) and (10) to complete Table II we may now construct Table III from Eq. (5). In the last three entries of Table III one or both of the dipole components is missing, so that two of these formulas are only partial mixtures while the last is pure quadrupole. However, they are included for the sake of completeness. In order to fill out Table III we can employ two auxiliary transformations. To find $I\Sigma(M,M')$ from $I\Sigma(M',M)$, we use

$$(\alpha,\beta,a,b,\varphi)\leftrightarrow(\alpha',\beta',a',b',\varphi'), \quad (14)$$

that is, exchange corresponding primed and unprimed quantities. To find $I\Sigma(-M,\mp M')$ from $I\Sigma(M,\pm M')$, we use

$$(a,a',b,b',\varphi,\varphi')\leftrightarrow(a,a',-b,-b',-\varphi,-\varphi') \quad (15)$$

with the Euler angles unchanged. These transformations may be proved by writing out Eq. (5) for the quantities involved and using Eqs. (9) and (10) to transform one into the other. The transformation $\gamma\leftrightarrow\gamma'$ is not included in Eq. (14) since this angle does not appear in the final result $I\Sigma$.

We may also compute the intensities given in Table IV from the first and third entries in Table III by using the upper signs and letting $a=a'$, $b=b'$, $\alpha=\alpha'$, and $\beta=\beta'$. This gives us $I_{21}(0)$ and $I_{21}(+1)$ when we take the square root. If we use transformation (15) to find $I\Sigma(-1,\mp1)$, we may then obtain $I_{21}(-1)$ by the same procedure. Similarly, from the last entry in Table III we obtain $I_2(\pm2)$ as in Eq. (6).

Finally, we note from Eq. (3) that electric and magnetic multipoles differ only in the sign of $\hat{\eta}_{-1}$. As a result of this, Tables II, III, and IV are the same for a magnetic quadrupole-electric dipole mixture as for the

TABLE III. Transmission factors $I\Sigma = |\mathcal{E}_{21}^*(M)\cdot\mathcal{E}_{21}'(M')|^2$.

M	M'	$I\Sigma$
0	0	$\sin^2\beta \sin^2\beta' \{[(3aa' \cos\beta \cos\beta' + bb' \cos(\varphi-\varphi'))\cos(\alpha-\alpha') + \sqrt{3}(ab' \sin\varphi' \cos\beta - ba' \sin\varphi \cos\beta')\sin(\alpha-\alpha')]^2$ $+[\sqrt{3}(ab' \cos\varphi' \cos\beta + ba' \cos\varphi \cos\beta')\sin(\alpha-\alpha') + bb' \sin(\varphi-\varphi')\cos(\alpha-\alpha')]^2\}$
±1	0	$\frac{1}{2}\sin^2\beta' \{[(\sqrt{3}aa' \cos\beta' \cos2\beta + ab' \cos\varphi' \cos\beta + \sqrt{3}ba' \cos\beta' \cos\varphi \pm bb' \cos(\varphi-\varphi')\cos\beta)\cos(\alpha-\alpha')$ $\pm(ab' \sin\varphi' \cos2\beta - \sqrt{3}ba' \cos\beta' \sin\varphi \cos\beta \mp bb' \sin(\varphi-\varphi'))\sin(\alpha-\alpha')]^2$ $+[(\sqrt{3}aa' \cos\beta' \cos\beta \pm ab' \cos\varphi' \cos2\beta \pm \sqrt{3}ba' \cos\beta' \cos\beta \cos\varphi + bb' \cos(\varphi-\varphi'))\sin(\alpha-\alpha')$ $+(-ab' \sin\varphi' \cos\beta + \sqrt{3}ba' \cos\beta' \sin\varphi \pm bb' \sin(\varphi-\varphi')\cos\beta)\cos(\alpha-\alpha')]^2\}$
1	±1	$\frac{1}{4}\{[aa'(\cos\beta \cos\beta' \pm \cos2\beta \cos2\beta') + ab' \cos\varphi'(\cos2\beta \pm \cos\beta \cos\beta')$ $+ba' \cos\varphi(\cos\beta \cos\beta' \pm \cos2\beta') + bb' \cos(\varphi-\varphi')(1 \pm \cos\beta \cos\beta')]\cos(\alpha-\alpha')$ $+[ab'(\cos\beta \pm \cos\beta' \cos2\beta) - ba' \sin\varphi(\cos\beta' \pm \cos\beta \cos2\beta') - bb' \sin(\varphi-\varphi')(\cos\beta \pm \cos\beta')]\sin(\alpha-\alpha')\}^2$ $+\frac{1}{4}\{[aa'(\cos\beta' \cos2\beta \pm \cos\beta \cos2\beta') + ab' \cos\varphi'(\cos\beta \pm \cos\beta' \cos2\beta)$ $+ba' \cos\varphi(\cos\beta' \pm \cos\beta \cos2\beta') + bb' \cos(\varphi-\varphi')(\cos\beta \pm \cos\beta')]\sin(\alpha-\alpha')$ $+[-ab' \sin\varphi'(\cos2\beta \pm \cos\beta \cos\beta') + ba' \sin\varphi(\cos\beta \cos\beta' \pm \cos2\beta')$ $+bb' \sin(\varphi-\varphi')(1 \pm \cos\beta \cos\beta')]\cos(\alpha-\alpha')\}^2$
±2	0	$\frac{1}{2}\sin^2\beta \sin^2\beta' \{[(\pm\sqrt{3}aa' \cos\beta \cos\beta' + ab' \cos\varphi')\cos(\alpha-\alpha') \pm ab' \sin\varphi' \cos\beta \sin(\alpha-\alpha')]^2$ $+[(\sqrt{3}aa' \cos\beta' \pm ab' \cos\varphi' \cos\beta)\sin(\alpha-\alpha') - ab' \sin\varphi' \cos(\alpha-\alpha')]^2\}$
2	±1	$\frac{1}{4}\sin^2\beta \{[(aa'\{\cos\beta' \pm \cos\beta \cos2\beta'\} + ab' \cos\varphi'\{\cos\beta \pm \cos\beta'\})\cos(\alpha-\alpha')$ $+ab' \sin\varphi'(1 \pm \cos\beta \cos\beta')\sin(\alpha-\alpha')]^2$ $+[(aa'\{\cos\beta \cos\beta' \pm \cos2\beta'\} + ab' \cos\varphi'\{1 \pm \cos\beta \cos\beta'\})\sin(\alpha-\alpha') - ab' \sin\varphi'(\cos\beta \pm \cos\beta')\cos(\alpha-\alpha')]^2\}$
2	±2	$\frac{1}{4}a^2a'^2 \sin^2\beta \sin^2\beta' [(\cos\beta \pm \cos\beta')^2 + \sin^2\beta \sin^2\beta' \cos^2(\alpha-\alpha')]$

electric quadrupole-magnetic dipole mixture we have been describing, although only the latter is of much physical interest.

EXPERIMENTAL POSSIBILITIES

Angular correlation measurements have been frequently used to determine the mixing ratio a/b and the relative phase φ.[3] At first[4] allowance was made for the possibility that the ratio is a complex number, $a/(be^{i\varphi})$. However, Lloyd[5] showed that the assumption of time-reversal invariance limits φ to the values 0 or π. Since the discovery of violations of the validity of parity conservation, attention has been turned toward experimental methods of checking time-reversal invariance too.[6–8] More recently, the Mössbauer effect has been proposed[9] as a technique for polarizing the daughter nucleus in an angular correlation experiment involving time-reversal and parity, and has been used[10] in a coincidence experiment to determine the $E2/M1$ mixing ratio of the 123-keV transition in Fe^{57}.

The results of our paper might be used to determine a/b and φ for nuclei which show the Mössbauer effect[11] and are known to emit mixed $E2/M1$ radiation.[12] Although only a limited number of such nuclei are known, and the Mössbauer effect requires the ground state to be the final state of a low-energy (<150-keV) transition, still Zeeman experiments using only the Mössbauer effect can serve as a complement to the techniques described above. Of particular interest would be a more accurate check of time-reversal invariance for such nuclei.

CONCLUSION

The theory of dipole-quadrupole mixtures has been presented in detail for emitter and absorber nuclei oriented in magnetic fields so that separated Zeeman lines appear. The method used is a traditional one since it amounts to an extension of Malus' law, discussed in most texts on optics. Both $\mathcal{E}$ (the "polarizer") and $\mathcal{E}'$ (the "analyzer") are projections of the electric vectors on the plane of observation, the $\hat{\eta}_1$, $\hat{\eta}_{-1}$ or $\hat{\imath}$, $\hat{\jmath}$ plane. Malus' $\cos^2(\alpha-\alpha')$ law holds for the case of plane-polarized radiation, $M=M'=0$. It is obtained by projecting one vector on the other, squaring the magnitude

TABLE IV. Intensities for quadrupole-dipole mixtures.

$$I_{21}(0) = (3/4)a^2 \sin^2 2\beta + b^2 \sin^2\beta$$
$$I_{21}(\pm1) = \tfrac{1}{2}[a^2(\cos^2\beta + \cos^2 2\beta) \pm 2ab \cos\varphi(\cos^2\beta + \cos2\beta) + b^2(1+\cos^2\beta)]$$
$$I_2(\pm2) = (1/8)a^2(4\sin^2\beta + \sin^2 2\beta)$$

[3] For example, T. Tamura and H. Yoshida, Nucl. Phys. **30**, 579 (1962).
[4] D. S. Ling, Jr., and D. L. Falkoff, Phys. Rev. **76**, 1639 (1949).
[5] S. P. Lloyd, Phys. Rev. **81**, 161 (1951).
[6] M. Morita and R. S. Morita, Phys. Rev. **107**, 1316 (1957); **110**, 461 (1958).
[7] E. M. Henley and B. A. Jacobsohn, Phys. Rev. **113**, 225 (1959).
[8] B. A. Jacobsohn and E. M. Henley, Phys. Rev. **113**, 234 (1959).
[9] M. Morita, Phys. Rev. **122**, 1525 (1961).
[10] H. de Waard and F. van der Woude, Phys. Rev. **129**, 1342 (1963).
[11] Third International Conference on the Mössbauer Effect, edited by A. J. Bearden, Rev. Mod. Phys. **36**, 496 (1964).
[12] G. N. Belozerskii and Yu. A. Nemilov, Usp. Fiz. Nauk **72**, 433 (1960) [English transl.: Soviet Phys.—Usp. **3**, 813 (1961)].

of the result and dividing by the intensities as in Eq. (8). We have extended this method to include elliptically polarized radiation for multipole mixtures and the pure multipoles which are special cases of these mixtures. Since Eqs. (3) and (13) are perfectly general, the method may be extended to multipole mixtures of any order. Possible use of these results in experiments has also been briefly described.

PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Beta Decay of Y^{90m}†

PHILIP W. DAVIS, JEAN KERN,* AND RAYMOND K. SHELINE
Department of Physics, Florida State University, Tallahassee, Florida
(Received 14 May 1964)

An 0.620-MeV β^- branch has been observed to compete with the previously reported gamma decay from the 3.14-h Y^{90m}. The calculated values, derived from the shell model for the branching ratio [(0.620-MeV β^-)/(0.482-MeV γ) $=1.16\times10^{-3}$] and the log*ft* (7.54) are compared with the experimental values of $3.8\pm1\times10^{-3}$ and 7.04 ± 0.13, respectively. The small discrepancy is probably due to impurities in the shell-model configurations assumed for the transition.

INTRODUCTION

THE 0.685-MeV ${}_{39}Y_{51}{}^{90m}$ level ($T_{1/2}=3.14$ h) has been shown[1–4] to gamma decay to an 0.203-MeV level and then to the ground state in a simple cascade with no measurable crossover. Spins and parities of 7+ for the 0.685 MeV level and 3− for the 0.203-MeV level have been established by the above groups, the ground-state spin and parity being previously established as 2− by Bartholomew.[5] From shell-model considerations, the 39th proton is in the $p_{1/2}$ shell and the 51st neutron is in the $d_{5/2}$ shell outside of a $g_{9/2}$ closed shell of 50 neutrons.[6] This implies that the 2− ground state and the 3− state result from the $(p_{1/2}d_{5/2})$ configuration. The simplest assumption as to the configuration of the 7+ isomeric level is to promote one proton into the $g_{9/2}$ shell, creating the $(g_{9/2}d_{5/2})$ configuration, and allowing the gamma decay from this state to involve a change only in the state of one nucleon.

The levels in ${}_{40}Zr_{50}{}^{90}$ have been studied by Ford,[7] Sheline,[8] Lazar *et al.*,[9] Bjørnholm *et al.*,[10] and Bayman *et al.*[11] In his study of this nucleus, Ford[7] has shown that the low-lying states in Zr^{90} should be determined by the proton configurations $(p_{1/2})^2$, $(g_{9/2})^2$, and $(p_{1/2}g_{9/2})$. Sheline[8] first observed the low-lying expected levels of $(1.752\text{ MeV})_{0+}$, $(2.182\text{ MeV})_{2+}$, and $(2.315\text{ MeV})_{5-}$ by populating them through the decay of Nb^{90}. Due to its spin, the 5− level can be unambiguously assigned to the $(p_{1/2}g_{9/2})$ orbital. Experimentally, this level was found to decay 84% to the ground state with an $E5$ gamma and 14% to the 2.182-MeV level by an $E3$ gamma transition. Furthermore, it was shown[8–10] that the ground state and the first excited state should both be mixtures of $(g_{9/2})^2$ and $(p_{1/2})^2$ configurations.

The relative population of the ground and first excited 0+ states in Zr^{90} by the Y^{90} ground state[12] through β^- decay and by the Zr^{90} $(2.182)_{2+}$ state through γ decay, establishes[8,10,11] that the ground-state configuration is $63\%\ (p_{1/2})^2+37\%(g_{9/2})^2$.

THEORETICAL

By examining the initial and final configurations of the states involved in the beta transition between the ground states of Y^{90} and Zr^{90}, it can be seen that this transition can be described as the transformation of a $d_{5/2}$ neutron into a $p_{1/2}$ proton. In a similar fashion, the $(g_{9/2}d_{5/2})_{7+}$ isomeric state in Y^{90} could be expected to decay into the $(p_{1/2}g_{9/2})_{5-}$ excited state in Zr^{90} by a $d_{5/2}$ neutron transforming via a new beta transition into a $p_{1/2}$ proton.

Not only will it be reasonable to expect that the log*ft* value for the two beta decays should be similar, but that one should be able to predict the log*ft* value of the new transition by using the log*ft* value of the ground-state transition after including the percentage (63%) of mixing of the ground-state configuration in Zr^{90} and a geometrical factor. The geometrical factor is needed

† This work was supported in part by the U. S. Atomic Energy Commission under Contract AT-(40-1)-2434.
* Supported in part by the "Fonds National Suisse de la Recherche Scientifique."
1 R. L. Heath, J. E. Cline, C. W. Reich, E. C. Yates, and E. H. Turk, Phys. Rev. **123**, 903 (1961).
2 C. Carter-Waschek and B. Linder, Nucl. Phys. **27**, 415 (1961).
3 W. S. Lyon, J. S. Eldridge, and L. C. Bates, Phys. Rev. **123**, 1747 (1961).
4 W. L. Alford, D. R. Koehler, and C. E. Mandeville, Phys. Rev. **123**, 1365 (1961).
5 G. A. Bartholomew, P. J. Campion, J. W. Knowles, and G. Manning, Nucl. Phys. **10**, 590 (1959).
6 P. F. A. Klinkenberg, Rev. Mod. Phys. **24**, 63 (1952).
7 K. W. Ford, Phys. Rev. **98**, 1516 (1955).
8 R. K. Sheline, Physica **23**, 923 (1957).
9 N. H. Lazar, G. D. O'Kelley, J. H. Hamilton, L. M. Langer, and W. G. Smith, Phys. Rev. **110**, 513 (1958).
10 S. Bjornhølm, O. B. Nielsen, and R. K. Sheline, Phys. Rev. **115**, 1613 (1959).
11 B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev. **115**, 1627 (1959).
12 O. E. Johnson, R. G. Johnson, and L. M. Langer, Phys. Rev. **98**, 1517 (1955).